1894MNRAS..54..289I

If Professor Newcomb can *prove* that with the dual systems of time-measurement in use in practical astronomical work the equation (1) is inexact and (2) is exact, then there is an end of my explanation of the cause of the change of the residual differences (Tabular R.A. from Hansen—Observed R.A. of Moon) about the year 1864, which is evident when the quantities $\alpha_{\prime}^{s}$ computed from the equation (2) are adopted as the tabular right ascensions for the meridian transits. But, on the other hand, if my proofs of the equation (1) are sound, the explanation of this *per saltum* change is shown conclusively to result from the differences between the quantities $\alpha^{s} - \alpha_{\prime}^{s}$ before and after 1864, which directly result from the different values of the constants l and N employed in the formation of the solar tables in use before and after 1864.

The discussion of the residuals (Hansen—Observed) is not appealed to as a proof of the truth of the equation (1), which must be, and has been, independently proved; but it is certainly no disproof to show that if we neglect the corrections $\delta l^{s} + 86400^{s} \,.\, xr$ and use the quantities $\alpha_{\prime}^{s}$ for the tabular right ascensions at the meridian transits of the Moon, the residuals which should remain sensibly unchanged about 1864, if the neglect of these corrections is a legitimate one, are found to have changed to such an extent that the same mean motions will not represent the observations before and after that date, and that these *per saltum* changes disappear when the tabular right ascensions are computed from equation (1).

Tables to Facilitate the Application of Gauss's Method of Computing Secular Variations. By R. T. A. Innes.

In Vol. I. of Newcomb's Astronomical Papers of the American Ephemeris, Professor G. W. Hill suggested the computations of the functions $\chi(\tau)$ and $\psi(\tau)$, both functions of θ.

If we take another function a related to θ as follows:—

$$\frac{1-a}{1+a} = \sqrt{\frac{\cos\left(60^{\circ}+\frac{\theta}{3}\right)}{\cos\frac{\theta}{3}}}.$$

M. O. Callandreau has published tables to six places of $\log m^{4} n^{4} \chi(\tau)$ and $\log \dfrac{\psi(\tau)}{\chi(\tau)}$ in vol. xviii of *Les Annales de l'Observatoire de Paris* for 580 values of a from 0·000 to 1·000, its extreme limits.

I had, previously to seeing M. Callandreau's tables, com-

1894MNRAS..54..289I

menced calculating log M and log N, using θ as the argument. They are connected with Hill's functions as follows :—

$$M = \left[\frac{\cos\left(60^\circ - \frac{\theta}{3}\right)}{\cos\theta}\right]^{-2} \psi(\tau)$$

$$N = \frac{\psi\tau}{\chi\tau}.$$

I believe such tables of log M and log N will be useful to astronomers as a check to M. Callandreau's. They also avoid the calculation of the intermediate function a.

Most of the values of log M and log N given in the tables herewith were computed with seven-figure logarithms, so that the last figure is uncertain. I have arranged the tables in the form—

$$f(\theta + h) = f(\theta) + h\frac{df}{d\theta} + h^2 \frac{1}{2}\frac{d^2f}{d\theta_2}.$$

Thus to compute log N for $-52^\circ\ 32'\ 36''$, which we may write $-52^\circ - {}^\circ{\cdot}5433$ or $-53^\circ + {}^\circ{\cdot}4567$

− 52°	gives	+0·1381654	− 53°	gives	+0·1385790
−·5433	,,	+ 2278	+·4567	,,	− 1864
+·295	,,	+ 16	+·21	,,	− 11
		+0·1383916			+0·1383915

The differences from 60° to 90° of θ are not given; they will be required but rarely, and M. Callandreau's tables are to be preferred for high values.

The numerical application of Hill's formulæ suggests a few modifications.

We may divide A, B, and C by a^2; the value of θ will be unchanged. V may be equal to

$$p + q\frac{\psi(\tau)}{\chi(\tau)}.$$

Then q may be removed from F_1, F_2, and F_3.

In F_1 and J_1 write $\left(\frac{a_1}{a}\right)^2$ or α^2 for $\frac{1}{a^2}$, a being the disturbing planet. From F_3, J_3, and $\frac{a}{\tau} W_0$ the factor sin I may be removed to the final equations. From $\frac{a}{\tau} R_0$, $\frac{a}{\tau} S_0$, and $\frac{a}{\tau} W_0$ the factor

$$\left(\frac{a_1}{a}\right)^2 \cos^2\phi$$

may be likewise removed to the end.

1894MNRAS..54..289I

We have now

$$\frac{a}{r}\mathrm{R}_0 = \left(\frac{r}{a}\right) q^{-\frac{7}{2}} \left[\frac{\cos\left(60^\circ - \frac{\theta}{3}\right)}{\cos\theta}\right]^2 \mathrm{M}(\mathrm{F}_1 + \mathrm{J}_1\mathrm{V})$$

with corresponding equations for $\frac{a}{r}\mathrm{S}_0$ and $\frac{a}{r}\mathrm{W}_0$.

Not the least merit of Gauss's methods consists in its numerous checks. Besides the even and odd functions agreeing in the totals, as pointed out by Professor Hill, we get from

$$\left[\frac{da}{dt}\right] = 0$$

the valuable check

$$\mathrm{B}_0^{(c)} = -\frac{1}{2}\mathrm{A}_1^{(s)} \chi \tan\phi.$$

Also as A, B sin ϵ, B cos ϵ, J_1, &c., can be made explicit functions of sin E, cos E, &c., the sum total of values can be ascertained at the beginning, and will prevent systematic errors. One example will suffice :—

$$\mathrm{B}\sin\epsilon = -\left[k_1\frac{a_1}{a} e \cos\phi_1 \sin \mathrm{K}_1\right] + \mathrm{C}_1 \cos \mathrm{E} + \mathrm{C}_2 \sin \mathrm{E}_1 ;$$

and as the sums of cosines and sines for any even regular division of the circumference is nil, if we take n divisions, the sum total of B sin ϵ will be

$$-n\left[k_1\frac{a_1}{a} e \cos\phi_1 \sin \mathrm{K}_1\right].$$

Gauss's method will also give the perturbations independent of the position of the disturbing planet. In the case of the Earth disturbed by *Venus* we have approximately

$$\frac{d\phi}{dt} = \frac{m_1 n}{1+m}\left(\frac{a_1}{a}\right)^2 \cos^2\phi_1 \cos\phi \sin \mathrm{E} \,.\, \frac{a}{r}\mathrm{R}_0 ;$$

$$\int\frac{d\phi}{dt}\,dt = \frac{-m}{1+m}\left(\frac{a_1}{a}\right)^2 \sin 1'' \,\mathrm{A}_0^{(c)} \sin(l'' - \varpi'') ;$$

$$\delta\phi = 0''{\cdot}908 \sin(l'' - \varpi''). \qquad [\text{Le Verrier} = 0''{\cdot}907.]$$

1894MNRAS..54..289I

θ	Log M	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$	Log N	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$
° −90	9·5573194	− 0	− 38	0·1461280	− 0	−55
−89	9·5573156	− 77	− 38	0·1461225	− 110	−55
−88	9·5573041	− 153	− 38	0·1461060	− 220	−55
−87	9·5572849	− 230	− 38	0·1460784	− 331	−55
−86	9·5572581	− 306	− 38	0·1460398	− 441	−55
−85	9·5572237	− 383	− 38	0·1459902	− 551	−55
−84	9·5571816	− 460	− 38	0·1459296	− 662	−55
−83	9·5571318	− 536	− 38	0·1458579	− 772	−55
−82	9·5570743	− 613	− 38	0·1457752	− 882	−55
−81	9·5570092	− 690	− 38	0·1456815	− 992	−55
−80	9·5569363	− 767	− 38	0·1455768	− 1102	−55
−79	9·5568559	− 843	− 38	0·1454610	− 1213	−55
−78	9·5567677	− 920	− 39	0·1453342	− 1323	−55
−77	9·5566718	− 997	− 39	0·1451964	− 1433	−55
−76	9·5565682	− 1074	− 39	0·1450476	− 1544	−55
−75	9·5564569	− 1152	− 39	0·1448877	− 1654	−55
−74	9·5563378	− 1229	− 39	0·1447168	− 1764	−55
−73	9·5562111	− 1306	− 39	0·1445348	− 1875	−55
−72	9·5560766	− 1384	− 39	0·1443419	− 1985	−55
−71	9·5559343	− 1461	− 39	0·1441379	− 2095	−55
−70	9·5557843	− 1539	− 39	0·1439228	− 2205	−55
−69	9·5556265	− 1617	− 39	0·1436968	− 2316	−55
−68	9·5554609	− 1695	− 39	0·1434597	− 2426	−55
−67	9·5552875	− 1773	− 39	0·1432116	− 2536	−55
−66	9·5551063	− 1851	− 39	0·1429524	− 2647	−55
−65	9·5549173	− 1929	− 39	0·1426822	− 2757	−55
−64	9·5547204	− 2008	− 39	0·1424010	− 2867	−55
−63	9·5545157	− 2087	− 39	0·1421087	− 2978	−55
−62	9·5543031	− 2165	− 39	0·1418054	− 3088	−55
−61	9·5540826	− 2244	− 40	0·1414911	− 3199	−55
−60	9·5538542	− 2324	− 40	0·1411657	− 3309	−55
−59	9·5536179	− 2403	− 40	0·1408293	− 3419	−55
−58	9·5533736	− 2483	− 40	0·1404818	− 3530	−55
−57	9·553121	− 2562	− 40	0·1401233	− 3640	−55
−56	9·5528612	− 2642	− 40	0·1397538	− 3750	−55
−55	9·5525929	− 2722	− 40	0·1393733	− 3861	−55
−54	9·5523167	− 2803	− 40	0·1389817	− 3971	−55
−53	9·5520324	− 2883	− 40	0·1385790	− 4082	−55

1894MNRAS..54..289I

θ	Log M	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$	Log N	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$
−52°	9·5517400	− 2964	− 40	0·1381654	− 4192	−55
−51	9·5514395	− 3045	− 40	0·1377407	− 4302	−55
−50	9·5511309	− 3127	− 41	0·1373049	− 4413	−55
−49	9·5508141	− 3208	− 41	0·1368582	− 4523	−55
−48	9·5504892	− 3290	− 41	0·1364004	− 4633	−55
−47	9·5501561	− 3372	− 41	0·1359315	− 4744	−55
−46	9·5498147	− 3455	− 41	0·1354517	− 4854	−55
−45	9·5494651	− 3537	− 41	0·1349608	− 4964	−55
−44	9·5491072	− 3620	− 42	0·1344588	− 5074	−55
−43	9·5487410	− 3703	− 42	0·1339459	− 5184	−55
−42	9·5483665	− 3787	− 42	0·1334220	− 5295	−55
−41	9·5479834	− 3871	− 42	0·1328870	− 5405	−55
−40	9·5475922	− 3956	− 42	0·1323410	− 5515	−55
−39	9·5471924	− 4040	− 42	0·1317840	− 5625	−55
−38	9·5467841	− 4125	− 42	0·1312160	− 5735	−55
−37	9·5463674	− 4210	− 43	0·1306369	− 5845	−55
−36	9·5459420	− 4296	− 43	0·1300469	− 5955	−55
−35	9·5455081	− 4382	− 43	0·1294459	− 6065	−55
−34	9·5450656	− 4468	− 43	0·1288339	− 6175	−55
−33	9·5446144	− 4555	− 44	0·1282109	− 6284	−55
−32	9·5441545	− 4643	− 44	0·1275770	− 6394	−55
−31	9·5436858	− 4730	− 44	0·1269321	− 6504	−55
−30	9·5432084	− 4818	− 44	0·1262762	− 6614	−55
−29	9·5427221	− 4907	− 44	0·1256093	− 6723	−55
−28	9·5422270	− 4996	− 44	0·1249316	− 6832	−55
−27	9·5417229	− 5085	− 45	0·1242429	− 6941	−55
−26	9·5412099	− 5175	− 45	0·1235433	− 7051	−55
−25	9·5406879	− 5266	− 45	0·1228327	− 7160	−55
−24	9·5401568	− 5356	− 46	0·1221113	− 7269	−54
−23	9·5396166	− 5448	− 46	0·1213790	− 7378	−54
−22	9·5390673	− 5540	− 46	0·1206357	− 7487	−54
−21	9·5385087	− 5632	− 46	0·1198817	− 7595	−54
−20	9·5379409	− 5725	− 46	0·1191167	− 7704	−54
−19	9·5373638	− 5818	− 47	0·1183410	− 7812	−54
−18	9·5367773	− 5912	− 47	0·1175544	− 7920	−54
−17	9·5361814	− 6007	− 47	0·1167571	− 8027	−54
−16	9·5355760	− 6102	− 48	0·1159490	− 8135	−54
−15	9·5349610	− 6197	− 48	0·1151301	− 8243	−54

1894MNRAS..54..289I

θ	Log M	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$	Log N	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$
$\circ$ −14	9·5343364	−6293	−48	0·1143005	−8350	−54
−13	9·5337022	−6390	−49	0·1134601	−8457	−54
−12	9·5330583	−6488	−49	0·1126091	−8564	−53
−11	9·5324045	−6586	−49	0·1117474	−8670	−53
−10	9·5317410	−6686	−50	0·1108751	−8776	−53
− 9	9·5310674	−6785	−50	0·1099922	−8882	−53
− 8	9·5303839	−6885	−51	0·1090987	−8988	−53
− 7	9·5296902	−6986	−51	0·1081946	−9093	−52
− 6	9·5289865	−7088	−51	0·1072800	−9198	−52
− 5	9·5282726	−7191	−52	0·1063549	−9303	−52
− 4	9·5275483	−7294	−52	0·1054195	−9407	−52
− 3	9·5268137	−7399	−52	0·1044736	−9510	−52
− 2	9·5260686	−7504	−53	0·1035174	−9614	−52
− 1	9·5253130	−7609	−53	0·1025508	−9717	−52
0	9·5245468	−7716	−53	0·1015739	−9820	−51
+ 1	9·5237699	−7823	−54	0·1005869	−9922	−51
+ 2	9·5229823	−7932	−54	0·0995895	−10024	−51
+ 3	9·5221837	−8040	−55	0·0985821	−10124	−50
− 4	9·5213742	−8151	−55	0·0975647	−10224	−50
+ 5	9·5205536	−8262	−56	0·0965374	−10324	−50
+ 6	9·5197219	−8373	−56	0·0954999	−10424	−50
+ 7	9·5188789	−8487	−57	0·0944526	−10522	−49
+ 8	9·5180245	−8601	−57	0·0933955	−10620	−49
+ 9	9·5171588	−8715	−58	0·0923286	−10717	−48
+10	9·5162815	−8831	−58	0·0912521	−10813	−48
+11	9·5153925	−8949	−59	0·0901659	−10909	−48
+12	9·5144917	−9067	−59	0·0890704	−11004	−47
+13	9·5135791	−9186	−60	0·0879652	−11098	−47
+14	9·5126545	−9307	−61	0·0868508	−11190	−46
+15	9·5117177	−9429	−61	0·0857271	−11283	−46
+16	9·5107687	−9552	−62	0·0845942	−11374	−45
+17	9·5098073	−9676	−62	0·0834523	−11464	−45
+18	9·5088334	−9802	−63	0·0823014	−11553	−44
+19	9·5078469	−9929	−64	0·0811418	−11640	−44
+20	9·5068476	−10057	−64	0·0799734	−11727	−43
+21	9·5058355	−10186	−65	0·0787964	−11813	−43
+22	9·5048104	−10318	−66	0·0776110	−11896	−42
+23	9·5037720	−10450	−66	0·0764173	−11979	−41

1894MNRAS..54..289I

θ	Log M	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$	Log N	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$
+24°	9·5027204	−10584	− 67	0·0752152	−12060	−40
+25	9·5016552	−10720	− 68	0·0740053	−12139	−39
+26	9·5005764	−10857	− 69	0·0727875	−12217	−39
+27	9·4994838	−10996	− 70	0·0715619	−12293	−38
+28	9·4983773	−11136	− 71	0·0703288	−12368	−37
+29	9·4972565	−11279	− 71	0·0690884	−12441	−36
+30	9·4961215	−11422	− 72	0·0678407	−12511	−35
+31	9·4949721	−11568	− 73	0·0665861	−12580	−34
+32	9·4938080	−11716	− 74	0·0653247	−12647	−33
+33	9·4926290	−11865	− 75	0·0640567	−12711	−32
+34	9·4914349	−12016	− 76	0·0627825	−12773	−30
+35	9·4902257	−12170	− 77	0·0615022	−12833	−29
+36	9·4890009	−12327	− 78	0·0602160	−12889	−28
+37	9·4877604	−12484	− 79	0·0589243	−12944	−27
+38	9·4865041	−12643	− 80	0·0576272	−12996	−25
+39	9·4852317	−12806	− 82	0·0563251	−13044	−24
+40	9·4839429	−12970	− 83	0·0550183	−13090	−22
+41	9·4826377	−13137	− 84	0·0537070	−13133	−20
+42	9·4813155	−13307	− 86	0·0523918	−13171	−19
+43	9·4799762	−13479	− 87	0·0510728	−13208	−17
+44	9·4786196	−13654	− 88	0·0497504	−13239	−15
+45	9·4772454	−13832	− 90	0·0484251	−13266	−13
+46	9·4758533	−14012	− 91	0·0470972	−13290	−11
+47	9·4744430	−14196	− 92	0·0457671	−13309	− 9
+48	9·4730142	−14382	− 94	0·0444354	−13324	− 6
+49	9·4715666	−14572	− 96	0·0431023	−13334	− 4
+50	9·4700998	−14765	− 97	0·0417686	−13338	− 1
+51	9·4686136	−14961	− 99	0·0404348	−13339	+ 2
+52	9·4671077	−15160	−101	0·0391009	−13332	+ 4
+53	9·4655815	−15365	−103	0·0377683	−13319	+ 7
+54	9·4640347	−15572	−105	0·0364371	−13302	+11
+55	9·4624671	−15783	−107	0·0351078	−13278	+14
+56	9·4608781	−15998	−109	0·0337816	−13246	+18
+57	9·4592675	−16217	−111	0·0324587	−13208	+21
+58	9·4576346	−16442	−113	0·0311401	−13162	+25
+59	9·4559793	−16669	−116	0·0298263	−13107	+30
+60	9·4553008	−16904	−118	0·0285187	−13043	+34

1894MNRAS..54..289I

θ	Log M	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$	Log N	$\frac{d}{d\theta}$	$\frac{1}{2}\frac{d^2}{d\theta^2}$
+61°	9·4525986	...	...	0·0272177		
+62	9·4508724	...	...	0·0259244		
+63	9·4491215	...	...	0·0246398		
+64	9·4473455	...	...	0·0233648		
+65	9·4455436	...	...	0·0221007		
+66	9·4437154	...	...	0·0208485		
+67	9·4418600	...	...	0·0196097		
+68	9·4399769	...	...	0·0182853		
+69	9·4380653	...	...	0·0171772		
+70	9·4361245	...	...	0·0159863		
+71	9·4341535	...	...	0·0148149		
+72	9·4321517	...	...	0·0136632		
+73	9·4301180	...	...	0·0125364		
+74	9·4280515	...	...	0·0114334		
+75	9·4259511	...	...	0·0103575		
+76	9·4238157	...	...	0·0093111		
+77	9·4216442	+	...	0·0082976		
+78	9·4194350	...	...	0·0073169		
+79	9·4171869	...	...	0·0063752		
+80	9·4148983	...	...	0·0054748		
+81	9·4125674	...	...	0·0046197		
+82	9·4101922	...	...	0·0038142		
+83	9·4077706	...	...	0·0030629		
+84	9·4053004	...	...	0·0023717		
+85	9·4027783	...	...	0·0017468		
+86	9·4002011	...	...	0·0011964		
+87	9·3975647	...	...	0·0007299		
+88	9·3948632	...	...	0·0003601		
+89	9·3920884	...	...	0·0001062		
+90	9·3892283	...	...	0·0000000		

Sydney: 1893 *June* 19.